Alternative Research Metaphors and the Social Context of Mathematics Teaching and Learning*

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My point of view toward mathematics and mathematics teaching is undoubtedly colored by my background I hated arithmetic, but I liked algebra because I could do it better than most of my fellow students, and I loved geometry for itself

I took my first degree in physics because I was a natural born tinkerer and mathematics had no lab or computers, in those days, to provide an external source of insight. Besides, the mathematics that was taught in the small liberal arts college I attended was clearly seen as subservient to physics; namely, the calculus, advanced calculus, and differential equations. There was little reason for me to suspect it was the queen of the sciences. I took my graduate degrees in science education, having abandoned physics in the middle of quantum mechanics whose differential equations I couldn't solve — perhaps, as I thought, because I couldn't visualize the physical system They seemed to work for others like *magic*, and *that* is what I didn't like. But education was, and still is, searching for foundations: philosophy (but philosophy of science was where the action was), psychology (but cognitive psychology was where the action was), and sociology (but linguistics and ethnography were where the action was — and still is). Clearly, I wanted "action" — something to compete with physics which I had abandoned. Or had it abandoned me? I still have hope it will return to Einstein's view that being able to compute psi square is not enough

I joined mathematics education formally with my second university post, in 1962, as Research Coordinator for Max Beberman's second UICSM math project — the one that produced "Shrinkers and Stretchers," "Motion Geometry," and "Vector Geometry" (The third one, on elementary school math, died when Max died in England some ten years ago)

What I admired in Max was that he wanted children to think, to discover mathematical ideas and patterns, and to learn to think mathematically. In a mathematics classroom, he was a master craftsman! Given a complete mathematical analysis of the curriculum which Herb Vaughan, Bernie Friedman (whose version of applied math turned out to be as pure as Vaughan's) and other mathematicians taught him, he could bring it "back to life" in the classroom While working with Max, I became acquainted with the two upper right-hand boxes in the diagram reported last year from the CMESG. (Figure 1)

Max Beberman had only a few apprentices. I had been an apprentice in '59 - '60 during the first UICSM project. Alice Hart, Eleanor McCoy and perhaps a half-dozen others were

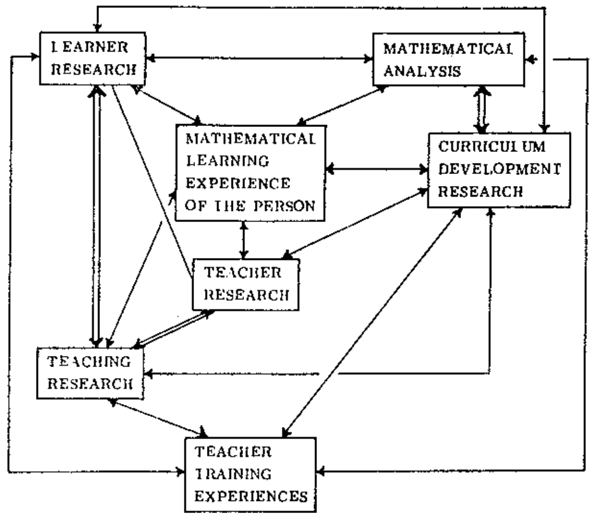

Figure 1

much better apprentices — totally dedicated to absorbing his classroom technique for years or even decades. I wasn't dedicated to mastering his craftsmanship. I had found out what he did in 9th grade algebra, and I learned how to do it myself. I wanted to know why it was possible psychologically, but I also wanted to master teacher training in a more direct way than Max himself had done. I was convinced it was possible to train large numbers of expert practitioners of the math teacher's craft, but Max's teachers' institute wasn't succeeding in doing that.

I was also fascinated by the critics whom Max regularly invited to visit the project: Morris Kline (who never came), Alexander Wittenberg, George Polya, and Imre Lakatos (whom I invited) Their arguments (and demonstration classes in Wittenberg's case) were very moving. But I believed Max was right — you couldn't construct a formal pedagogy out of their non-mechanical, intuitive view of mathematics. (It took a mechanical, i.e , dead, analysis build on sets, Gentzen's natural deduction [1969], or the like, to produce clear principles of classroom practice). If only most math teachers were amateur mathematicians, reading Martin Gardner's column in *Scientific American* regularly, trying out his puzzles, being taken in by his April Fool jokes and sheepishly spending the next few months working out how they got taken in!!! *Then* they'd know about mathematical intuition without principles of pedagogy and recognize it when they saw it in pupils (Or would they?)

I became convinced in the early sixties that the way Max's teaching worked could be better understood by means of Piaget's research. Piaget, it seemed, had the only

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psychology of mathematics and physics going — but I was later persuaded I was wrong. Piaget wasn't dealing with psychology at all, but philosophy — anti-positivist, anti-empiricist, anti-realist, above all anti-phenomenologist epistemology. Psychology was only a sort of weapon with which to fight Carnap, Popper, Russell, Husserl and Merleau-Ponty [see especially Piaget, 1969]. He flaunted his genetic fallacy (genetic epistemology) in the face of the *criterion* philosophers, and he undermined the *experience* philosophers with his cognitive structures. I shifted my attention to the two blocks in the upper left of Figure 1.

Piaget appealed to mathematicians and logicians to help him discover how mathematics was biologically possible and was intrigued with the first, and the so-called "second", Erlangen programs [Bourbaki, 1939]. In the "first", Klein saw geometry as emerging from the underlying structures of transformation groups, and in the "second" MacLane saw algebra as emerging from categories. If transformation groups and categories had biological foundations, mathematics was biologically possible.

Piaget's stance on mathematics education eventually became quite clear in the paper he circulated at Exeter (1975). He endorsed the "new math" as a content field within which teacher and pupil should work, but he objected to formal methods of mathematical work. His approach really didn't seem to help very much in seeing how to alleviate the dismal state of mathematics teaching K-12 [Stake and Easley, 1978]. My problem was, what kind of theory is needed on which to found a pedagogy of mathematics? No answer was forthcoming, but the narrowness of most research in math and science education appalled me, and I tried to prepare my doctoral students for a "no-holds barred" attack on this problem. We needed to be able to understand why things went right and wrong, explain to teachers what puzzled them, and provide a conceptual framework against which they could work on their own problems, pedagogical ones. The first part of what follows illustrates my quest for such positive foundations and the second part questions it all.

(For the bibliophile, among doctoral students I've influenced in some way are, Hiroshi Ikeda, Robert Comley, and John Clement (at the University of Massachusetts). Dan Knifong, George Shirk, Stan Erlwanger, produced notable math education dissertations. Ted Nicholson, and Bernadine Stake and Shirley Johnson now have math related dissertations in progress and Linda Brandau and James Kau are doing preliminary doctoral studies on the social process of math teaching under my direction.)

In 1977, I published a paper, from which the table of Figure 2 comes. I don't want to discuss that paper except to say that I didn't find it very helpful in guiding my research. I wrote it to call the attention of my students (in several content fields) to metaphors implicit in most educational research. I'm still working on the problem it raised of how to discover and choose better ones. However, I do want to say something about that table that's not in the paper. There is an order — perhaps a partial order — in the table concerning the proportion of formal and informal research being

KEY METAPHORS	Combinations 1 Mixtures	Sampling 2 Dice	Feedback 3 Codes Thermostat	Games 4 Poker	Operations Criteria 5 Consistency	Syntax 6 Language Games	Organic Structures 7 "Experts"
Typical Questions:	What % variance is accounted for by these variables?	What is probability of this result?	What is the means-ends program?	What are the rules the strategies?	What are the criteria?	What is the "logic" of "teaching," "learning," etc.?	What are the structures, how do they work?
Typical Solutions to:							
1. What makes t good at teaching x to y?	Interaction of t variables with y variables	Arranged or chance matching of t variables with y variables	Efficient coding system	Creative "plays"	Consistent modelling of precise language	y agrees on language game with t	Good match between t and y structures
2. Why can't p learn to do q?	p ignores q variables	p's sample of q too small	p uses wrong codes	p is playing wrong game	Use of inadequate language	Confusion of multiple meanings	p assimilates lesson to wrong structures
3. Why can't t teach x to y better?	t ignores y variables	Arranged or chance mis-match of t with y variables	t has poor information on p	Poor sportsmanship, etc.	Non-critical thinking	Thinks words carry their meanings	Forced patterns of behavior, against p's structures
Uses of Knowledge	policy to control variables	avoid decisions based on poor samples	adjust system for efficient information processing	follow optimum strategy	use consistent language	use folk wisdom	look for opportunities to teach

Figure 2

done in each perspective (See Figure 3) By formal research, I mean that algorithms or mathematical theorems can be used; by informal, I mean they cannot be — at least, until they are developed The 7th perspective has the least use of mathematics, but still it has some Other metaphors are needed, and informality is to be expected with the new ones. (I would like to add two more metaphors: process and intuition, which have as yet been used mainly without mathematics. I'll say more about them later.)

Mainly, I observe that, with informal uses of metaphors, the choice of metaphor problem can be worked on, but formal methods tend to stop the search for better metaphors

Perhaps the most important creative work in research is done between successive studies; that's where metaphors can change That's what I want to try to share with you — the thoughts and feelings that occurred *between* some of the studies I've been involved in. Therefore, I won't give the research details that might be expected in a research summary.

The following problems have concerned me for a long time:

> What accounts for successful teaching of mathematics by the few?
>
> What accounts for unsuccessful teaching and learning of mathematics by the many?
>
> What could be done to remedy unsuccessful teaching and learning?

While I want to explore with you alternatives to those seven (widely recognized and utilized) modeling perspectives about which I wrote in 1977, I should say a little more about my disappointment in them

There is a powerful cognitive effect in using mainly just one or two of the established perspectives: you believe you are open to every possible question and every possible answer, yet you tend to ask and answer only those questions that come most naturally within that perspective You use the power of the models within the perspective you have but you don't choose a new perspective. For example, "Gestalt switches" between some of the key metaphors are very difficult I find myself unhappy in settling on any one of them because each one seems impotent to improve mathematics and science teaching. However, what many people want to know, as the cognitive effect predicts, is why you can't modify or adapt one of the standard approaches to solve the problems of practice — and the only answer I've come up with is that I have really tried doing this, and failed I'd like to show you how I tried as a practitioner (teacher, tutor, teacher educator), and why I'm searching for new metaphors now

This is autobiographical, but that's not the point of it The items are selected to make the above point and not to say who I am. Many other workers could select research experiences of their own to make the same point. In fact, only in that way can each researcher check the meaning and validity of my conclusion for him or her

Starting with the first of the seven perspectives, the combinatorial or "mixture" conception, with Robert Conley [1965], we tried item-by-item comparison of student populations from different curricula on a standardized test. [Cronbach, 1964] Item profiles and matrix sampling don't tell you much — it's too easy to make up hypotheses to explain the differences observed between the two populations (see Figure 4a) Of course, this approach is more sensible than using test scores to compare students taking different curricula, when the test is, at best, related to just one of them. At any rate, we found few differences between the profiles for UICSM and traditional algebra classes — certainly not enough to justify the investment that had been made in developing the new curriculum.

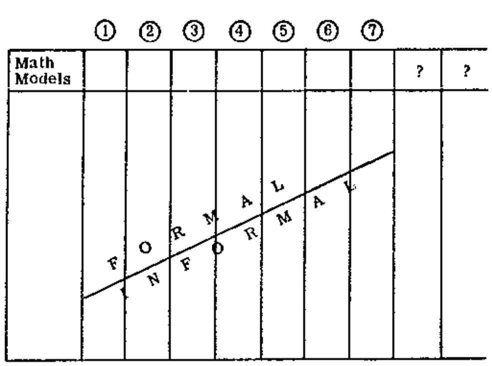

Figure 3

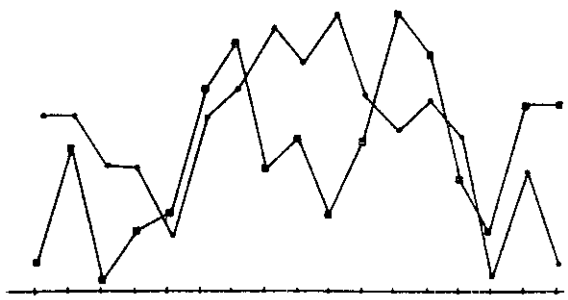

Figure 4a

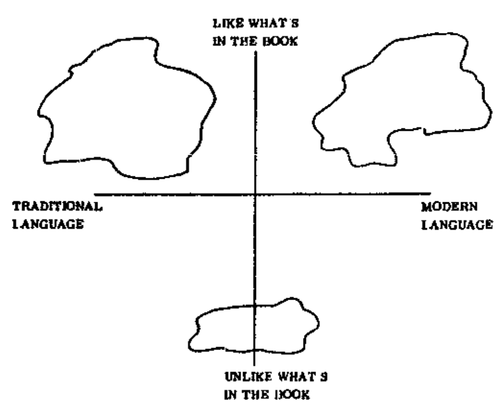

Figure 4b

Here the conception imposed by a profile is that of a mixture of different ingredients in different proportions. However, good math learning does not appear to be a particular mixture of certain "ingredients" It's more like a structure — more like a flower arrangement or a bonsai than a martini. At least, that's the way it feels to me.

Comparing populations of students and teachers, however, factor analysis by Hiroshi Ikeda [1965] helped to make some sense out of the observed mixtures What he found was that teacher preferences for our test items could be represented by the two bi-polar factors shown in Figure 4b. Ikeda's hypotheses was that teacher-held objectives matched what the pupils learned. However, the factor structure of teachers' preferences did not correlate significantly with that of class performance on the same items. So how do you improve math education? We had really tried to use the psychometric mixture metaphor creatively but made no real progress in a useful understanding.

Of course, any complex structure should have many profiles projected onto different sub-spaces. The choice of a space for a research study should be precisely determined by the inner form of the structuring, as a chemist does — think of DNA. However, without that guidance, the dimensions available are too rich an assortment to be searched through by trial and error until the structure is found. The inferences from test items to what concepts or attitudes are learned is much too "stretched out" to be made with confidence. This method, like other psychometric and statistical methods, is primarily useful now as a method of demonstration, i e., demonstrating whatever variance is accounted for by the items used One needs a prior conception in order to generate the items Today, with hidden curricula and alternative preconceptions better known, factor analysis seems even more unconvincing

Sequencing of problems

Teaching is linear in time — so there should be at least a partial order in the problems used in successful teaching that would be beneficial to other teachers. Such an order could be defined in terms of the properties of the problems used However teaching appears to be much more than sequencing problems in terms of their formal properties. Of course, there are well-known formal snags in most disciplines that can be avoided by proper preparation. On the whole, learning does not appear to be linear and incremental; rather, it seems to involve unpredictable insights, forgetting, and much relearning. [Denny, 1978]

Pattern recognition is a new metaphor, related to information processing that was also emphasized by Polya. Like many metaphors, it seems to work when it is promoted by a skillful teacher Could we understand it as a process in individual students? I wanted to try.

There'd been talk of "teachable moments". The idea occurred to me to capture them in terms of heart-rate increases, measured while students worked math problems on PLATO. I decided to try to find out if I could identify moments of insight in terms of the formal structure of problem solving using response times. (Of course, if we allow ample response times, it is possible for a student to give

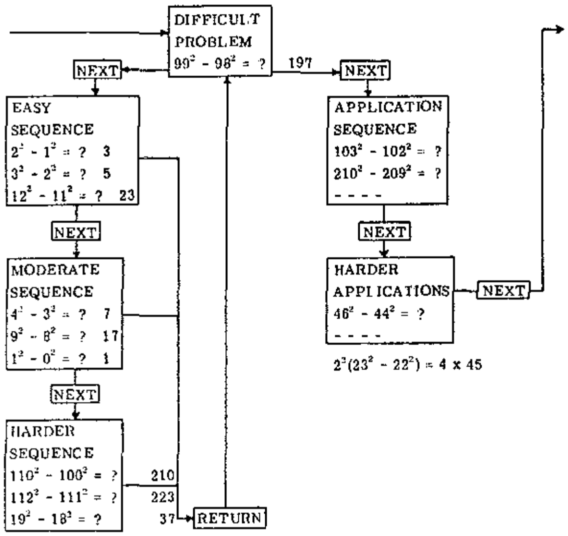

Figure 5

correct answers to all the problems in such a sequence without recognizing the pattern) Herb Wills, Al Avner, and Pat Cutler helped me devise problems [Wills, 1967], set up this program, and connect electrodes to the students chests and analyze the data. Problems were arranged on PLATO as illustrated in Figure 5 Mathematical insight was indicated by quick success on the difficult problem after guided progress through easier problems. Such insights were sometimes "hot" and sometimes "cool", as indicated by an increase or lack of increase of heart rate Moreover, there were some "hot" moments that were not related to any mathematical event. Clearly, this wasn't it! Something more than pattern recognition is going on in a class that becomes excited when they discover a pattern What else is going on besides individual mathematics? Something social must be going on. Perhaps teachers who are skillful at getting a class to make pattern discoveries exploit the emotional response of insightful students. So, it doesn't appear that pattern recognition is regularly exciting or motivating, but sometimes it clearly is

In Figure 6a, is depicted the kinetic energy of a pupil thinking out loud while working a math problem, as sound and movement energy might be recorded Figure 6b represents the kinetic space constrictions and expansions during the same time period. [Witz and Easley, 1976] This opens new doors conceptually, but does not help solve the problem of math teaching A new metaphor is mathematics as social control [see Stake and Easley, 1978, Ch. 16] In Figure 6c, a teacher-pupil dialog is interpreted figuratively in terms of classroom competition and social control Experienced teachers undoubtedly read the figurative meaning of classroom dialog and pay a lot of attention to it. Studying it helps researchers find out what else is going on in teacher-pupil interaction. Theoretically, it affects mathematical learning. But once anthropologists have learned to interpret the symbolic meanings of the classroom, can they be of any help to the improvement of

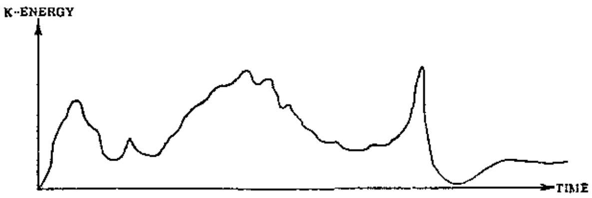

Figure 6a

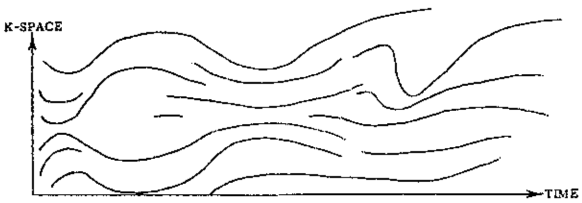

Figure 6b

Literal meanings	Figurative meanings
T: What s negative four, squared? (watches hands waving) Bill?	
Bill: That's easy, sixteen	[You ve got to make points when you can]
John: Positive sixteen	[Why d you call on him? I m smarter]
T: We agreed to that. What's negative four cubed? Mary?	[Cut it out you guys Everyone gets a turn]

Figure 6c

learning mathematics? Some say not. I observe that:

1 Observers are very hard to train for recording this kind of meaning It took me many years to get the feel of it and, even now, I have to concentrate very hard, or I slip back into the so-called literal code.
2 It's not yet clear what teachers get out of this kind of reporting i.e., there are only two or three teachers who have reported positive benefits — certainly not a representative group
3. It is far from clear, although promising, what pupils will get out of this research. Certainly, we have them in mind.

Back into the literal code! Can we increase information processing and crowd out symbolic meaning?

Working with PLATO had inspired the notion, an elaboration of the third perspective (Figure 2), that its data-retrieval and editing potential could be used to develop a man-machine system that would be self-improving (Figure 7a) and adaptive to the learning characteristics of students Starting with a curriculum system, I would augment the human intelligence that was involved by coupling it with a computer. This would be a McLuhan-esque extension of our brains and sense organs [See Easley, 1966] Don Bitzer, Kikumi Tatsuoka, Betty Kendzior, Al Avner and others helped It didn't work.

Teachers and authors were much too busy and self-directed to hook into a man-computer system. Although I myself had mastered the PLATO computer operations for searching for error patterns and detecting possible pupil misconceptions and applied them to lessons I wrote, I could only discover the most trivial misconceptions. In short, I couldn't use the system to improve it All improvements I made came from reflecting on the natural teacher-pupil interaction phenomena (Reports on this project [Easley, 1968, SIRA] were filed with the U.S Office of Education.)

This remains a problem today with PLATO and similar systems It was also a problem in the UICSM vector geometry curriculum project Formative evaluation of UICSM texts was spoiled by the slow process of data collection and analysis. Most ideas for improvement came from other sources. The big difficulty that took me years to recognize, is that learning how to teach mathematics is not the inductive, accumulative process all these models assume it is. It is some kind of constructive process The constructive metaphor is another reason to look to Piaget and the seventh perspective of Figure 2. Furthermore, learning about mathematics teachers should be looked at similarly, as I intimated at the end of my 1977 paper.

Papert has a version of Piaget's scheme metaphor in which the mind of the child is seen as a "room-full of experts" who respond independently to problems in terms of their expertise, but without any executive to decide which one responds to which problem [Papert, 1975] Bernardine Stake [1980] and I have been using this metaphor in studying counting by primary grade children. Why aren't children in primary grades more able to remember numbers they have just counted or computed? We looked at long-term and short-term memory What are children doing? Why? We observed that the process of counting is more important to most K-2 children than the result of counting Process is a key metaphor Here's where the dynamic processes of cognitive structures need development. Several schemes can be distinguished as involved in different aspects of counting. For example, for the first one depicted, we developed a theory of certain counting errors. Figure 7b depicts the problem of controlling the partition or envelope that separates what has been counted from what hasn't been The confirmation of a child's counting is illustrated as

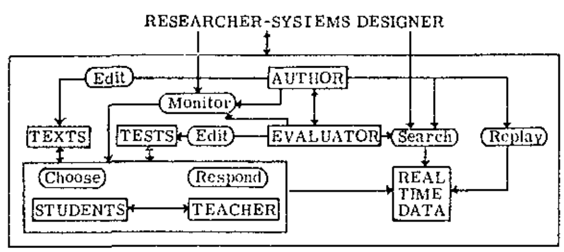

Figure 7a

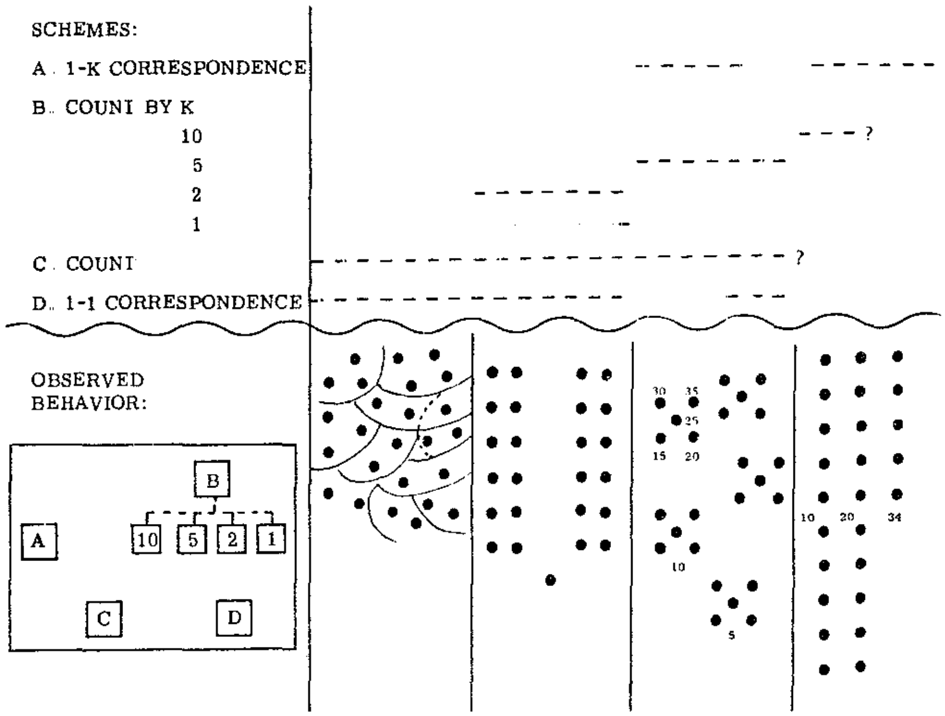

Figure 7b

a small-scale process in which a cluster of objects is subitized as containing 1,2,3 or 4 objects and enumerated with a matching cluster of numbers from the counting sequence. The problem of how to control the spreading envelope that separates what has been counted from what hasn't been is not often addressed, in our experience, by primary grade children. The confirmation or checking process is located at the "micro" level rather than the "macro" level where the result occurs and where we had been looking. This theory suggests posing counting problems for K-1 children to work on. If teachers could be persuaded that skill in counting disorderly collections is *not* an objective for which all children should be taught a solution, then counting techniques wouldn't need to be taught them, and they could learn to enjoy solving problems by themselves early on. Perhaps the habit would stick.

Some answers pupils give to some problems are so clearly not derivable from pedagogical processes that to account for them, it is necessary to bring in intuition as another metaphor. Easley and Zwoyer [1975] placed a lot of hope in the metaphor of "teaching by listening". Just listening closely to a pupil's erroneous explanation, often seems to stimulate the student's incubation of richer, better understandings. Several points need to be made:

1. Probing, in a clinical interview, is an art needed to expose students' ideas.
2. Listening is difficult when you are concentrating on what you want to teach.
3. Counter examples can help clarify students' ideas, if one has listened well.
4. The student often straightens things out a day or so after he/she has tried hard to explain things to a good listener.

When Bernadine Stake and I interviewed primary grade students on mathematical concepts on successive days, they nearly always improved the second day. Figure 8 illustrates

S. $1.06 > 1.50 > .79 > .4 > 002$

I. Explain it. What about the first two?

S. That's just the way it is. 1.06 is bigger than 1.50.

I. What if you had 1.60?

S. That's the same thing as 1.06

You see, $.06 = \frac{6}{10}$ and $.60 = \frac{60}{100}$

and $\frac{60}{100} = \frac{30}{50} = \frac{6}{10}$ So, $.06 = .60$ and $1.06 = 1.60$

I. What if you had $1.60 and I had $1.06, who'd have more?

S. I think you're trying to kill me with money.
Let's see, with money it's different. It's the way you say it,
you'd have a dollar and six cents and I'd have a dollar and 60 cents.
But with gallons or liters, you'd have a fraction, and they're
the same.

I. So what does .002 mean?

S. $\frac{2}{10}$

I. .4?

S. $\frac{4}{10}$

I. .79?

S. $\frac{79}{100}$

Figure 8

a confusion which a 5th grade student straightened out by himself within a week after this interview.

Working in the seventh perspective, Klaus Witz and I solved the problem of identification of cognitive structures [Witz, 1973; Easley, 1974] We showed problems that are created for test item-analysis and profile interpretations by the alternative structures a child would likely have available to use in responding to a single item But our interview methods can't be used by many teachers in the classroom — probing is usually not possible.

Bernadine Stake and I decided to work on teacher's problems, in hope they'd use what we found. For example, these: Why does counting conflict with place value? Why do K-1 children say 2+3=3 or 5−3=3?

Teachers do a lot of piloting of calculation processes. Why is so little significant struggling with conceptual frameworks going on in classrooms? Teachers try to make math easier and wind up making it harder later on. "Piloting" is Lundgren's metaphor [1979] I want to study piloting sympathetically Structural reconstruction of children's ideas really doesn't help many teachers (even very good ones) because they have their own "agenda" Consider the dialog in Figure 9 (abstracted from several times this much material in Rosalind Driver's [1973] dissertation). The teacher's intervention clearly interrupts the student's discussion The teacher and the students are operating in two incompatible framework The teacher automatically reduces everything to measurable variables, and the students employ concepts of resistance in which whether or not resistance is a variable quality for a given table and whether it is measurable or not are precisely the points of their argument. Teachers and pupils often tolerate this pluralism with mutual respect and get along fine.

Bob Stake and I [1978] published ethnographic portrayals of what math teachers are teaching. [See also Easley, 1979] We found they teach:

a) personality and personal values by exhortation and example
b) group control (without either grouping by ability or isolating the slow, difficult and destructive)
c) socialization: trying hard
   following directions
   following rules
   admitting errors
   neatness
   submitting to drill
   hard work
   memorizing
d) values: "Do your own work"

We also found that teachers:

a) have problems understanding forgetting and learning,
b) cry for help in adapting math materials to children of different backgrounds from their own,
c) believe that math supervisors, teacher educators, and administrators can't help much because they're not even considering how to use math to control and socialize children.

For example Figure 10a depicts the closing moves of a game of "guess my rule" as played by a 7th grade pre-algebra class [Stake and Easley, 1978, Ch 16] The teacher

S_1: Well, I say that when you put more weight on the table there s no more resistance It just sits there.

S_2: But, you see, it s got to resist more to hold up the larger weight.

S_1: Don't be silly, the table's just there in the way

I: (dropping by) What variables have you been measuring?

S_2: Well we haven t actually measured the weights yet, we re waiting for a scale

I: What else are you going to measure?

S_1: How high up the spring holds it

I: Oh, the length of the spring. Anything else?

S_2: Oh, the diameter of the wire

I: Here, you can use that scale. (leaves)

S_1: The table s not doing anything.

S_2: If you make the weight big enough, it ll do something It ll smash.

S_1: Well, if you let an elephant stand on it, sure! Then it doesn t resist anymore.

S_2: Look (places a meter stick so half of it extends off the table), when you put a weight on there, it will bend--so you know how much it pushes back.

Figure 9

GUESS MY RULE: PRE-ALGEBRA, 7th GRADE

$\triangle$	$\square$
1	0
2	3
3	8
4	15

$(\triangle \times \triangle) - 1 = \square$

$\triangle$	$\square$
4	1
9	2
$\cancel{5}$	

$\sqrt{\triangle} - 1 = \square$

$\triangle = \bigcirc \times \bigcirc$

$\bigcirc - 1 = \square$

I: They have to keep trying!

Figure 10a

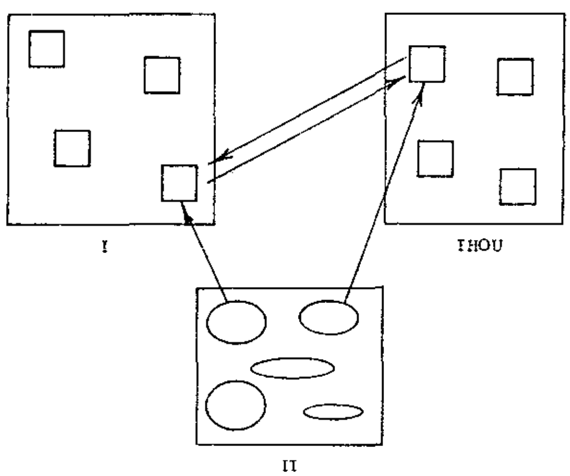

Figure 10b

watches while the class guesses the rule of one pupil, in the process generating the table on the left and the equation underneath. When the second table is set up by the winner of the previous game, the teacher intervenes, calling first one pupil and then another to the board to guess entries for the left column. The two are stumped because their entries are not accepted (they are not perfect squares), and they refuse to continue guessing — to the teacher's great consternation. Later, she explains: "They have to keep trying!" The teacher's dramatic take-over of a student game is somewhat softened when she tries to write the equation without using the radical sign — since it has not yet been introduced.

The observer helps her out with the suggestion that she write two equations — which she does at the right. Her prime agenda are clear! She wants to shape all of her pupils into persons who'll cope well with algebra in terms of their personal style or manifest attitude. School mathematics is moral training, I concluded.

How does this picture help? To many, it is a picture of despair. No one group of people (teachers, math specialists, administrators) can do anything much without the others getting involved. The whole "eco-system" has to be examined by its own elements. (We have to discover the enemy and recognize it as us.) Old pedagogical metaphors get buried in mathematical research methods — but that's the wrong way to do applied math. Fresh conceptions are needed that are sufficiently close to the target practitioners (teacher educators, teachers themselves, or students) that they can recognize how to express their problems and can (or could) participate in shaping the research. Then they won't see research as a monstrous distortion, casting their problems in a foreign metaphor — or if they do, and they have the courage to say so — they will recognize, as some researchers don't, that no particular metaphor is a necessary one. It is truly an "I, Thou, It" situation. [Hawkins, 1969] In Figure 10b I attempt to represent that different aspects of the "It" are interpreted in terms of different metaphors in the minds of "I" and "Thou".

Whether multiple metaphors can help is a good question. Perhaps, it may be argued, the real problem cannot be fully represented in any one metaphor. It is difficult for me to presuppose a real world in which the real problem exists objectively. An alternative we can consider is that the problem exists in the constructed world of the practitioner's experience, and it is that experience that holds together the multiple images of several metaphors. Lakatos [1976] implicitly has shown that within the Eulerian research program of his *Proofs and Refutations*, unlike a Kuhnian stable period of normal science, inquiry moves across several different metaphors of polyhedra. Should we expect less of mathematics practitioners and researchers?

Considered strategically, the first main point is that such informal methods as clinical interviews and participant observations have convinced me that children and teachers alike have ideas I wouldn't have suspected, which *if ignored* may lead them to mistrust their own responses to various educational programs. As a teacher educator, I risk putting teachers down if I ignore their ideas but demonstrate to them how they can use their students' ideas. Can I work in a way that shows the same respect for teachers' ideas as I have for students' ideas? My concern for the damage they may be doing to students' own self-confidence may so intrude in my relation with teachers that I will spoil their self-confidence and the opportunity for real communication with them.

The second point is: the more I study math teachers, at any level, the more convinced I become that mathematics is so embedded in social interaction — where they have their own sense of how the game is to be played and what standards of justice they are to maintain — that I am forced to enter into a dialog with them about ethics, personal values, and culture and help them struggle with issues they are even less prepared to face. Where do either of us get support for rethinking the ethics and sociology of math teaching and learning? Is equity the only issue? Is social stratification of society in terms of mathematical ability wrong? Shouldn't those in responsible technical positions: doctors, lawyers, engineers, etc. be required to have superior abstract reasoning ability?

The third point is that a very special design is needed to open up free communication between groups of math educators taking different perspectives. Here are three possibilities:

1. CAMP MEETINGS (Communication Across Multiple Perspectives)

identification of all viewpoints studied with mastery to criterion of two new ones by every participant

Video Teachers
Tapes Specialists

2. LONGITUDINAL CASE STUDIES of teachers, students representing different perspectives.

3. Put downs are so highly likely that an advocacy system for teachers may be needed.

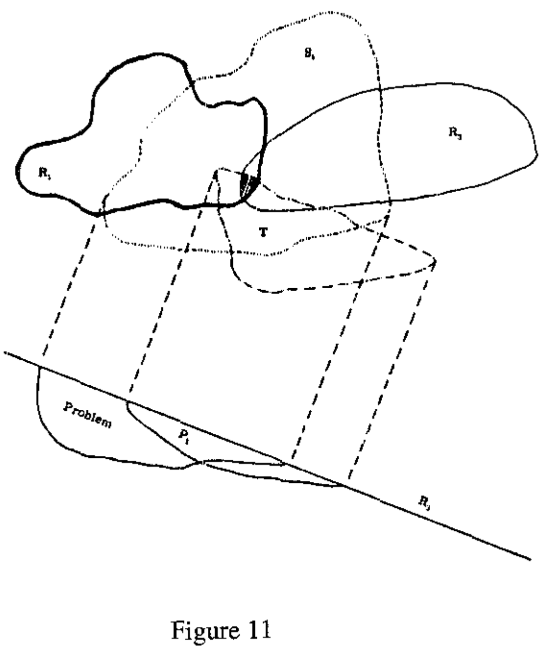

Figure 11

The primary need is to represent to researchers and curriculum specialists the problems that really concern teachers. That means representing problems in their own frameworks, for the frameworks researchers are accustomed to use really distort problems and make the results useless for most teachers (See Figure 11.) To do that, informal methods are required, by definition (except possibly for those few teachers with standard research training).

Clinical interviews with teachers and participant-observer case studies are required to capture their view of their problems, and some integration during investigations of such problems is needed to lead toward solutions they will recognize as worthy of their attention. It seems to be an act of faith that a teacher's problem solved will represent a step toward real improvements for student's mathematical learning. Of course, solving one problem often exposes another more serious one. But failure to solve genuine problems is only to leave teachers with those problems stranded and unable to reach any breakthroughs.

Conclusion

Even in the most educationally valuable classrooms, there is a significant amount of missed communication that would shock the teachers, if it were demonstrated to them. That is, as teachers, we always think we understand a good deal more than we really do of what our students are thinking. I believe that for all teachers, clinical interviews and careful analysis of video-tapes from the classroom can reveal major conceptual differences between their students' ideas and what they thought their students' ideas were. In that sense, the teacher in *Proofs and Refutations* represents a false ideal. The reason is simple: the dialog is a rational reconstruction of what is, to a significant extent, an irrational process. That is, the missed communications enhance the struggles of the students' ideas to sort themselves out organically and to find their own expressive metaphors, just because this process has been encouraged in the classroom. Unfortunately, that kind of teaching and learning happens rarely because expression of deviant ideas is so rarely accepted, and it cannot be significantly increased except by using it as the means of teacher education and, recursively, the means of the education of teachers of teachers, including the education of educational researchers, etc., etc., etc. The bright hope of the future lies only in the enormous generative power of branching recursive processes. Such a process connects the way we do research along many paths with what teachers and children do in classrooms.

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